Linear Quadratic Regulator (LQR)

$0 = P\check{A} + A^T P + \check{Q} - P\check{B}\check{R}^{-1}\check{B}^T P$

$u = -Kx = -R^{-1}B^T P x(t)$

eig($\dot{x} = Ax + Bu = (A - BK)x$

```
[K,P,p_cl] = lqr(A,B,Q,R)
[P1,K1,p_cl1] = icare(A,B,Q,R,[],[],[])
```

Pole Placement Design Using State Feedback

$\zeta = \dfrac{-\ln(PO/100)}{\sqrt{\pi^2 + \ln^2(PO/100)}}$ $\omega_n \approx \dfrac{4}{T_s \cdot \zeta}$

Poles $-\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$ $\omega_n = \dfrac{\pi}{T_p\sqrt{1-\zeta^2}}$

$\Delta_d(s) = (s-\mu_1)(s-\mu_2)(s-\mu_3)$

$\phi(A) = \phi(s=A)$

$= A^3 + 14A^2 + 60A + 200I_3$ $\phi(s) = \Delta_d(s)$

$$K = [0 \quad \cdots \quad 0 \quad 1]U^{-1}\Delta_d(A)$$

$|s\mathbf{I} - \mathbf{A} + \mathbf{BK}|$ ← For SYMS

$\dot{z} = (A - BK)z$ $u = -Kx$

$$L = \Delta_d(s)|_{s=A} V^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$|sI - (A - LC)| = 0$ ← For SYMS

$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{x}} \end{bmatrix} = \underbrace{\begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix}}_{A^*} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} r$$

Realizations of LTI Systems

$$\min_{x,\lambda} H(x,\lambda) := L(x) + \lambda^T f(x)$$

Controllability in Canonical Forms

$U = [B \quad AB \quad A^2B]$

rank(U)

$|sI - A| = a_2 \quad a_1 \quad a_0$

$M = \begin{bmatrix} a_1 & a_2 & 1 \\ a_2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$P = UM$

```
u=[B A*B A^2*B]  %ctrb(A,B)
rank(u)
syms s
det(s*eye(3)-A)
a=poly([1 2 3]) %poly([roots])=
% =[a3(1) a2(2) a1(3) a0(4)]
m=hankel([a(3), a(2), a(1)])
p=u*m
```

$\bar{A} = P^{-1}AP$ $\bar{B} = P^{-1}B$

$\bar{C} = CP$

Observability in Canonical Forms

$V = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$

$P = (MV)^{-1}$

```
syms s
det(s*eye(3)-A)
a=poly([1 2 3]) %poly([roots])=
% =[a3(1) a2(2) a1(3) a0(4)]
m=hankel([a(3), a(2), a(1)])
v=obsv(A,C)
rank(v)
pi=m*v
p= inv(pi)
```

Obtaining the State Equations

Without Derivatives of $u(t)$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & 1 & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-2} & -a_{n-1} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_0 \end{bmatrix}$$

$$C = [1 \quad 0 \quad \cdots \quad 0] \qquad D = 0$$

With Derivatives

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & 1 & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-2} & -a_{n-1} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$C = [-b_n a_0 + b_0 \quad -b_n a_1 + b_1 \quad \cdots \quad -b_n a_{n-1} + b_{n-1}]$$

$D = b_n$ n=order

$\dddot{y} + a_1\ddot{y} + a_2\dot{y} + a_3 y = b_0\dot{u} + b_1 u$ $\dfrac{b_0 s + b_1}{s^3 + a_1 s^2 + a_2 s + a_3}$

State-Space to a Transfer Function

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$$

Controllable Kalman Decomposition

$P = U = [B \quad AB \quad A^2B]$

$P = [c_1 \quad c_2 \quad \cdots \quad c_\mu \mid c_{\mu+1} \quad \cdots \quad c_n]$

From U Appended vectors

$$\bar{A} = \begin{array}{c} \begin{bmatrix} A_c & A_{12} \\ 0 & A_{\bar{c}} \end{bmatrix} \begin{array}{l} \mu \\ n-\mu \end{array} \\ \begin{array}{cc} \mu & n-\mu \end{array} \end{array}$$

$\bar{A} = P^{-1}AP$

$\bar{B} = P^{-1}B$

$\bar{C} = CP$

```
u=ctrb(A,B)
rank(u)
p=u
p(:,3)=[0 1 0]'
rank(p)
```

Observable Kalman Decomposition

$$P^{-1} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_\gamma \\ - \\ r_{\gamma+1} \\ \vdots \\ r_n \end{bmatrix}$$

From V

Appended

$\bar{A} = P^{-1}AP$

$\bar{B} = P^{-1}B$

$\bar{C} = CP$

```
v=obsv(A,C)
rank(v)   %/=n
Pinv=v
Pinv(3,:)=[0 1 0]
rank(Pinv)  %=n
```

Nonhomogeneous Solution

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$$

$$y(t) = Ce^{At}x(0) + C\int_0^t e^{A(t-\tau)}Bu(\tau)d\tau + Du$$

$x(t) =$

```
a=[]
syms s x1 x2 x3
x=[x1 x2 x3]';
ilaplace(inv(s*eye(3)-a))*x
```

+

```
a=[] b=[]
syms s x1 x2 x3 tau t
x=[x1 x2 x3]';
e=ilaplace(inv(s*eye(3)-a),t-tau)*x
int(jacobian(e,[x1 x2 x3])*b,tau)
```

$y(t) =$

```
a=[] b=[] c=[]
syms s x1 x2 x3
x=[x1 x2 x3]';
c*ilaplace(inv(s*eye(3)-a))*x;
```

```
C*expm(A*t)*x0  % or %
```

+

```
a=[] b=[]
syms s x1 x2 x3 tau t
x=[x1 x2 x3]';
e=ilaplace(inv(s*eye(3)-a),t-tau)*x
c*int(jacobian(e,[x1 x2 x3])*b,tau)
```